# Tom Carson · Bill Jordan Elementary and Intermediate Algebra

with Carson Math Study System

Fourth Edition

# **Your Learning Style**

To discover what kind of learner you are, complete the Learning Styles Inventory on page xiv (or in MyMathLab). Then in the textbook, watch for the Learning Strategy boxes and the accompanying icons that provide ideas for maximizing your own learning style.

#### **Your Learning Strategies**



#### Learning Strategy

Developing a good study system and understanding how you best learn is essential to academic success. Make sure you familiarize your-self with the study system outlined in the To the Student section at the beginning of the text. Also take a moment to complete the Learning Styles Inventory found at the end of that section to discover your personal learning style. In these Learning Strategy boxes, we offer tips and suggestions on how to connect the study system and your learning style to help you be successful in the course.

- 1. I remember information better if I write it down or draw a picture of it.
- 2. I remember things better when I hear them instead of just reading or seeing them.
- 3. When I receive something that has to be assembled, I just start doing it. I don't read the directions.
- 4. If I am taking a test, I can visualize the page of text or lecture notes where the answer is located.
- 5. I would rather have the professor explain a graph, chart, or diagram to me instead of just showing it to me.
- 6. When learning new things, I want to do it rather than hear about it.
- \_\_\_\_ 7. I would rather have the instructor write the information on the board or overhead instead of just lecturing.
- 8. I would rather listen to an audiobook than read the book.
- 9. I enjoy making things, putting things together, and working with my hands.
- 10. I am able to conceptualize quickly and visualize information
- 11. I learn best by hearing words.
- 12. I have been called hyperactive by my parents, spouse, partner, or professor.
- 13. I have no trouble reading maps, charts, or diagrams.
- 14. I can usually pick up on small such as like bells, crickets, or frogs or on distant sounds such as train whistles.
- \_\_\_\_ 15. I use my hands and gesture a lot when I speak to others.

### VISUAL AUDITORY TACTILE

#### Learning Strategy

In the To the Student section, we suggest that when taking notes, you use a red pen for definitions and a blue pen for rules and procedures. Notice that we have used those colors in the design of the text to connect with your notes. Definition Additive inverses: Two numbers whose sum in zero.

Examples of additive inverses: 15 and -15 because 15 + (-15) = 0 -9 and 9 because -9 + 9 = 00 and 0 because 0 + 0 = 0

Notice that 0 is its own additive inverse and that for numbers other than 0, the additive inverses have the same absolute value but *opposite* signs.

#### **Rules**

The sum of two additive inverse is zero.

The additive inverse of 0 is 0.

The additive inverse of a nonzero number has the same absolute value but opposite sign.

# **The Carson Math Study System**

The Carson Math Study System is designed to help you succeed in your math course. You will discover your own learning style, and you will use study strategies that match the way you learn best. You will also learn how to organize your course materials, manage your time efficiently, and study and review effectively.



**Fourth Edition** 

# **Elementary and Intermediate Algebra**

# **Tom Carson**

## **Bill Jordan**

Seminole State College of Florida

### PEARSON

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# Contents

Preface vi

To the Student xiii

Learning Styles Inventory xv

The Carson Math Study System xvii

#### Foundations of Algebra 1

- 1.1 Number Sets and the Structure of Algebra 2
- 1.2 Fractions 10
- 1.3 Adding and Subtracting Real Numbers; Properties of Real Numbers 20
- 1.4 Multiplying and Dividing Real Numbers; Properties of Real Numbers 34
- 1.5 Exponents, Roots, and Order of Operations 47
- 1.6 Translating Word Phrases to Expressions 58
- 1.7 Evaluating and Rewriting Expressions 65 Summary and Review Exercises 75 Practice Test 85

#### 2 Solving Linear Equations and Inequalities 87

- 2.1 Equations, Formulas, and the Problem-Solving Process 88
- 2.2 The Addition Principle of Equality 101
- 2.3 The Multiplication Principle of Equality 112
- 2.4 Applying the Principles to Formulas 125
- 2.5 Translating Word Sentences to Equations 130
- 2.6 Ratios and Proportions 138
- 2.7 Percents 150
- 2.8 Solving Linear Inequalities 162 Summary and Review Exercises 173 Practice Test 184 Chapters 1–2 Cumulative Review Exercises 187

## **3** Graphing Linear Equations and Inequalities 189

- 3.1 The Rectangular Coordinate System 190
- 3.2 Graphing Linear Equations 197
- 3.3 Graphing Using Intercepts 208

- 3.4 Slope–Intercept Form 217
- 3.5 Point-Slope Form 229
- 3.6 Graphing Linear Inequalities 239
- 3.7 Introduction to Functions and Function Notation 248 Summary and Review Exercises 263 Practice Test 275 Chapters 1–3 Cumulative Review Exercises 277

#### Systems of Linear Equations and Inequalities 279

- 4.1 Solving Systems of Linear Equations Graphically 280
- 4.2 Solving Systems of Linear Equations by Substitution; Applications 289
- 4.3 Solving Systems of Linear Equations by Elimination; Applications 300
- 4.4 Solving Systems of Linear Equations in Three Variables; Applications 312
- 4.5 Solving Systems of Linear Equations Using Matrices 321
- 4.6 Solving Systems of Linear Inequalities 330 Summary and Review Exercises 337 Practice Test 345 Chapters 1–4 Cumulative Review Exercises 347

#### 🗧 Polynomials 349

- 5.1 Exponents and Scientific Notation 350
- 5.2 Introduction to Polynomials 360
- 5.3 Adding and Subtracting Polynomials 371
- 5.4 Exponent Rules and Multiplying Monomials 380

- 5.5 Multiplying Polynomials; Special Products 390
- 5.6 Exponent Rules and Dividing

Polynomials 402 Summary and Review Exercises 416 Practice Test 423 Chapters 1–5 Cumulative Review Exercises 425

- Factoring 427
  - 6.1 Greatest Common Factor and Factoring by Grouping 428
  - 6.2 Factoring Trinomials of the Form  $x^2 + bx + c$  438
  - 6.3 Factoring Trinomials of the Form  $ax^2 + bx + c$ , where  $a \neq 1$  444
  - 6.4 Factoring Special Products 452
  - 6.5 Strategies for Factoring 460
  - 6.6 Solving Quadratic Equations by Factoring 464
  - 6.7 Graphs of Quadratic Equations and Functions 476 Summary and Review Exercises 486 Practice Test 493 Chapters 1–6 Cumulative Review Exercises 495

#### 7 Rational Expressions and Equations 497

- 7.1 Simplifying Rational Expressions 498
- 7.2 Multiplying and Dividing Rational Expressions 509
- 7.3 Adding and Subtracting Rational Expressions with the Same Denominator 519
- 7.4 Adding and Subtracting Rational Expressions with Different Denominators 525
- 7.5 Complex Rational Expressions 534
- 7.6 Solving Equations Containing Rational Expressions 542
- 7.7 Applications with Rational Expressions, Including Variation 551
   Summary and Review Exercises 564
   Practice Test 573
   Chapters 1–7 Cumulative Review Exercises 575

### O More on Inequalities, Absolute Value, and Functions 577

- 8.1 Compound Inequalities 578
- 8.2 Equations Involving Absolute Value; Absolute Value Functions 589
- 8.3 Inequalities Involving Absolute Value 595 Summary and Review Exercises 603 Practice Test 608 Chapters 1–8 Cumulative Review Exercises 610

### **9** Rational Exponents, Radicals, and Complex Numbers 612

- 9.1 Radical Expressions and Functions 613
- 9.2 Rational Exponents 623
- 9.3 Multiplying, Dividing, and Simplifying Radicals 633
- 9.4 Adding, Subtracting, and Multiplying Radical Expressions 641
- 9.5 Rationalizing Numerators and Denominators of Radical Expressions 648
- 9.6 Radical Equations and Problem Solving 658
- 9.7 Complex Numbers 667 Summary and Review Exercises 675 Practice Test 685 Chapters 1–9 Cumulative Review Exercises 687

### Quadratic Equations and Functions 689

- 10.1 The Square Root Principle and Completing the Square 690
- 10.2 Solving Quadratic Equations Using the Quadratic Formula 700
- 10.3 Solving Equations That Are Quadratic in Form 711
- 10.4 Graphing Quadratic Functions 720
- **10.5** Solving Nonlinear Inequalities **732**
- 10.6 Function Operations 743 Summary and Review Exercises 750 Practice Test 758 Chapters 1–10 Cumulative Review Exercises 761

#### **Exponential and Logarithmic** Functions 763

- 11.1 Composite and Inverse Functions 764
- 11.2 Exponential Functions 776
- 11.3 Logarithmic Functions 785
- 11.4 Properties of Logarithms 793
- 11.5 Common and Natural Logarithms 801
- **11.6** Exponential and Logarithmic Equations with Applications 806

Summary and Review Exercises 817 Practice Test 829 Chapters 1–11 Cumulative Review Exercises 831

- 12 Conic Sections 833 12.1 Parabolas and Circles 834

  - 12.2 Ellipses and Hyperbolas 847
  - 12.3 Nonlinear Systems of Equations 858
  - 12.4 Nonlinear Inequalities and Systems of Inequalities 864 Summary and Review Exercises 871 Practice Test 880 Chapters 1–12 Cumulative Review Exercises 882

### Elementary Algebra Review R-1

#### **Appendixes**

- A. Mean, Median, and Mode APP-1
- B. Arithmetic Sequences and Series APP-5
- C. Geometric Sequences and Series APP-12
- D. The Binomial Theorem APP-19
- E. Synthetic Division APP-24
- F. Solving Systems of Linear Equations Using Cramer's Rule APP-28

Photo Credits P-1

Collaborative Exercises c-1

Answers A-1

Glossary G-1

Index of Applications IA-1

Index I-1

# Preface

#### The Why behind Understanding Algebra

The Carson Math Study System with Learning Styles Inventory

#### **From the Authors**

Welcome to the fourth edition of *Elementary and Intermediate Algebra*! Revising our program has been both exciting and rewarding, and it has given us the opportunity to respond to valuable instructor and student feedback. With great pride, we share with you the improvements to this edition, as well as the key features and proven style of our approach.

Elementary and Intermediate Algebra, Fourth Edition, is one title in a series of four that also includes *Prealgebra*, Fourth Edition; *Elementary Algebra*, Fourth Edition; and *Intermediate Algebra*, Fourth Edition. We have designed our program to be versatile enough for use in a variety of teaching and learning formats, including standard lecture, selfpaced lab, hybrid, online, and even independent study.

We write in a relaxed, nonthreatening style, taking great care to ensure that students who have struggled with math in the past will be comfortable with our subject matter. Throughout the text, we explain *why* an algebraic process works the way it does, instead of just showing how to follow steps to solve problems. In addition, through problems from science, engineering, accounting, health, the arts, and everyday life, we link algebra to the real world.

Finally, to help students succeed in these courses, we offer the complete Carson Study System that includes a note To the Student, a Learning Styles Inventory, a Math Study System plan for developing a notebook as a personalized organizational and study tool, and integrated Learning Strategies from both the authors and students.

> Tom Carson Bill Jordan

#### The Carson Math Study System

The Carson Math Study System is designed to help students develop the skills (for example, time management, test prep, and note-taking) that students need to succeed in math, their college careers, and life.

- 1. **To the Student** (page xiii) focuses on why math is important and what students need to do to succeed in their math courses.
- 2. Taking the **Learning Styles Inventory** (page xv) will help students assess their particular style of learning and use that knowledge to identify helpful study skills.
- 3. The Math Study System (page xvii) offers students abundant suggestions for developing a class notebook to reflect their personal learning styles, taking effective notes, doing homework, and reviewing for quizzes and tests.
- 4. Throughout the text, **Learning Strategy** boxes offer advice on implementing the study system effectively based on a student's learning style (pages xv–xvi).

#### **New to this Edition**

In response to feedback from instructors and students, the presentation was refined; examples, exercises, Your Turns, and Instructor Notes were added; and real-data applications were updated or replaced with current data and topics. In addition, the order and content of some topics was improved for student comprehension and retention.

#### **Content Changes**

**Chapter 1** The objectives in Sections 1.3 and 1.4 were condensed. Instead of separate objectives for integers and rational numbers, each section has a single objective for rational numbers.

**Chapter 2** The former Chapter 3 on problem solving was streamlined and the topics integrated elsewhere. Former Sections 3.1 and 3.2 on ratios, proportions, and percents were moved to Chapter 2 as Sections 2.6 and 2.7. Former Sections 3.3–3.5 have been integrated into Chapter 4. Those sections involved solving problems with two unknowns using single-variable equations. In the authors' experience, however, students find it easier and more intuitive to solve those problems using systems of equations; so the topic is now covered with systems.

**Chapter 3** Graphing linear equations is now covered earlier, in Chapter 3.

**Chapter 4** Formerly Chapter 9, the chapter on systems of equations now follows the chapter on graphing linear equations. This keeps all of the discussion of linear equations together in a natural flow. This also introduces problems with two unknowns earlier in the text, using the more intuitive approach. Cramer's Rule has been moved to Appendix F.

**Chapter 7** The discussion of LCD and the examples of variation were streamlined and are now covered in a more visual presentation.

**Chapter 8** Function operations (formerly in Chapter 8) are now in Chapter 10, and the review section on graphing functions was deleted. The only functions that had not been introduced to this point were absolute value functions. Graphing absolute value functions is now an objective in Section 8.2 rather than in a separate section.

**Chapter 9** The development of complex numbers is now more precise.

**Chapter 10** Operations on functions are now covered in Section 10.6. The authors did this for two reasons. First, because all of the functions involved are polynomial functions, the topic fits better with the chapter on quadratic equations. Second, by placing this topic at the end of the chapter, it acts as a good transition into composite functions at the beginning of Chapter 11.

#### **New and Revised Features**

**Student Learning Strategies** and helpful tips written by successful students and recent college graduates were added throughout the text. These can be identified by the student's name at the end of the strategy.

The **Chapter Openers** now provide a brief topical overview of the chapter. In the instructor's edition, each opener now includes teaching suggestions for the chapter.

Each section now begins with **Warm-up Exercises** that review material presented previously and are helpful to understanding the concepts in the section.

Section Exercises are now grouped by objective to make it easier for students to connect examples with related exercises. In addition, new **Prep Exercises** help students focus on the terminology, rules, and processes corresponding to the exercises that immediately follow.

The **Chapter Summary and Review** is now interactive, with Review Exercises integrated within the summary to encourage active learning and review. For each key topic, students are prompted to complete the corresponding definitions, rules, and procedures.

Finally, we now offer two versions of MyMathLab: Standard MyMathLab courses allow instructors to build the course their way, offering maximum flexibility and control over all aspects of assignment creation. New Readyto-Go courses provide students with all the same great MyMathLab features, but make it easier for instructors to get started.

#### **Key Features**

In addition to the new and revised features just described, the following key components round out the comprehensive guided learning approach.

An Algebra Pyramid is used throughout the text to help students see how the topic they are learning relates to the big picture of algebra—focusing particularly on the relationship between constants, variables, expressions, and equations and inequalities (page 3). In the end-of-section Review Exercises, Chapter Review Exercises, and Cumulative Review Exercises, an Algebra Pyramid icon indicates the level of the pyramid that correlates to a particular group of exercises. This helps students determine what actions are appropriate with these exercises (for example, whether to "simplify" or "solve" (pages 74, 75–76, 187–188).

**Connection Boxes** help students understand how math concepts are related and build on each other (pages 190, 690).

Your Turn Practice Exercises, found after most examples, give students an opportunity to work problems similar to the examples they just saw. This practice step engages students and provides immediate feedback to help them develop confidence in their problem-solving skills (pages 2, 596).

**Real, Relevant, and Interesting Applications** in the examples and exercises reflect real-world situations in science, engineering, health, finance, the arts, and other areas. These applications illustrate the everyday use of basic algebraic concepts and encourage students to apply mathematical concepts to solve problems (pages 305–306, 559).

**Thorough Explanations** are key to student understanding. The authors take great care to explain not only *how* to do the math but also *why* the math works the way it does, how concepts are related, and how the math is relevant to students' everyday lives. Knowing all of this gives students a context in which to learn and remember math concepts.

### MyMathLab Ties the Complete Learning Program Together

#### MyMathLab® Online Course (access code required)

MyMathLab from Pearson is the world's leading online resource in mathematics, integrating interactive homework, assessment, and media in a flexible, easy-to-use format. MyMathLab delivers **proven results** in helping individual students succeed. It provides **engaging experiences** that personalize, stimulate, and measure learning for each student. And it comes from an **experienced partner** with educational expertise and an eye on the future.

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**Exercises:** The homework and practice exercises in MyMathLab are correlated to the exercises in the textbook, and they regenerate algorithmically to give students unlimited opportunity for practice and mastery. The software offers immediate, helpful feedback when students enter incorrect answers.

**Multimedia Learning Aids:** Exercises include guided solutions, sample problems, animations, videos, and eText access for extra help at point of use.

**Expert Tutoring:** Although many students describe the whole of MyMathLab as "like having your own personal tutor," students using My-MathLab do have access to live tutoring from Pearson, from qualified math instructors.

To help students achieve mastery, MyMathLab
can generate <b>personalized homework</b> based
on individual performance on tests or quizzes.
Personalized homework allows students to fo-
cus on topics they have not yet mastered. 🛛 🛌

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The **Adaptive Study Plan** makes studying more efficient and effective for every student. Performance and activity are assessed continually in real time. The data and analytics are used to provide personalized content reinforcing concepts that target each student's strengths and weaknesses.

#### Adaptive Study Plan powered by KNEWTON

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Announcements	Study Plan				Legend 🖨 🕐
Homework	You have earne	d 2 of 514 mastery points (MP),			View progress
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Resources	*	Name sets using the roster method.	Practice	Quiz Me	0 of 1 MP
	8.1	Basic Geometric Figures			
	*	Draw and name line segments, rays, and lines.	Practice	Quiz Me	0 of 1 MP

### Flexible Design, Easy Start-up, and Results for Instructors

Customize	d and Ready-to-Go Courses
course settings 🔯	MyMathLab*
modify 🗘 🚺	Section 2
Course Home	Module 2 Section 2: Solving Equations: The Multiplication Principle
Announcements	
Module 1	Read the <u>multimedia eText</u> .      Watch a video presentation.
▼ Module 2	Do Module 2 Section 2 Homework and Skills Maintenance assignments.
Section 1	
Section 2	
Section 3	
Mid-Module Review	
Section 4	

**Instructors can modify the left-hand nav and insert their own directions** onto course-level landing pages, and a **custom MyMathLab** course can be built to reorganize material and structure the course by chapters, modules, units—whatever the needs may be.

**Ready-to-Go courses** include pre-assigned homework, quizzes, and tests to make it even easier for instructors to get started.

The **comprehensive online gradebook** automatically tracks students' results on tests, quizzes, homework, and in the study plan. You can use the gradebook to intervene quickly if students have trouble or to provide positive feedback on a job well done. The data within MyMathLab is easily exported to a variety of spreadsheet programs such as Microsoft Excel. Instructors can determine which points of data to export and then analyze the results to determine success.

New features such as **Search/Email by criteria** make the gradebook a powerful tool for instructors. With this feature, instructors can easily communicate with both at-risk and successful students. Instructors can search by score on specific assignments, noncompletion of assignments within a given time frame, last log-in date, or overall score.



#### Additional Resources in MyMathLab

In addition to the robust course delivery, the course also include the *full Carson eText*, additional Carson Program Features, and the *entire set of instructor and student resources* in one easy place to access online.

For Students	For Instructors
<ul> <li>Student's Solutions Manual*</li> <li>JSBN-10: 0-321-92529-7</li> <li>JSBN-13: 978-03-21-92529-9</li> <li>Contains complete solutions to the odd-numbered section exercises and solutions to all of the section-level Review Exercises, Chapter Review Exercises, Practice Tests, and Cumulative Review Exercises, Practice Provides or with the MyMathLab access kit and includes the following resources for exchangeles with stepped-out solutions and similar Practice Exercises, keyed to the text by Learning Objective.</li> <li>References to textbook Examples and Section Lecture Videos for additional help.</li> <li>Additional Exercises with ample space for students to show their work, keyed to the text by Learning Objective.</li> <li>Video Resources</li> <li>Within MyMathLab, students can access short lectures for each section of the text, plus Chapter Test Prep Videos, which allow students to watch an instructor work through step-bystep solutions for all of the Chapter Test exercises from the textbook. All videos include optional English and Spanish captions.</li> <li>Trinted supplements are also available for separate purchase through MyMathLab, MyPearsonStore.com, or other retail outlets. They can also be value-packed with a textbook or MyMathLab code at a discount.</li> </ul>	<ul> <li>Annotated Instructor's Edition**</li> <li>JSBN-10: 0-321-92536-X</li> <li>JSBN-13: 978-0-321-92536-7</li> <li>Includes answers to all exercises, printed in bright blue near the corresponding problems.</li> <li>Useful teaching tips are printed in the margin.</li> <li>A * icon, found in the AIE only, indicates especially challenging exercises in the exercise sets.</li> <li>Instructor's Resource Manual with Tests and MiniLectures** (download only)</li> <li>JSBN-10: 0-321-92524-6</li> <li>JSBN-10: 0-321-92524-7</li> <li>A mini-lecture for each section of the text, organized by objective, includes key examples and teaching tips.</li> <li>Designed to help both new and adjunct faculty with course preparation and classroom management.</li> <li>Contains one diagnostic test per chapter; four freerresponse test forms per chapter, one of which contains higher-level questions; one multiple-choice test per chapter; a midchapter check-up for each chapter; one midterm exam; and two final exams.</li> <li>Instructor's Solutions Manual** (Download only)</li> <li>JSBN-10: 0-321-92525-4</li> <li>JSBN-10: 0-321-92525-4</li> <li>JSBN-10: 0-321-92525-4</li> <li>JSBN-13: 978-0-321-92525-1</li> <li>Ontains complete solutions to all even-numbered section exercises, Puzzle Problems, and Collaborative Exercises.</li> <li>PowerPoint® Lecture Slides** (download only)</li> <li>Present key concepts and definitions from the text.</li> <li>TestGen®</li> <li>Wew.pearsoned.com / testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank are available for download from Pearson Education's online catalog.</li> </ul>

To learn more about how MyMathLab combines proven learning applications with powerful assessment, visit **www.mymathlab.com** or contact your Pearson representative.

#### **Acknowledgments**

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> Tom Carson Bill Jordan

#### Why Do I Have to Take This Course?

Often, this is one of the first questions students ask when they find out they must take an algebra course. What a great question! But why focus on math alone? What about English, history, psychology, or science? Does anyone really use *every* topic of *every* course in the curriculum? Most jobs do not require that we write essays on Shake-speare, discuss the difference between various psychological theories, or analyze the cell structure of a frog's liver. So what's the point? The issue comes down to recognizing that general education courses are not job training. The purpose of those courses is to stretch and exercise the mind so that the educated person can better communicate, analyze situations, and solve problems, which are all valuable skills in life and any job.

Professional athletes offer a good analogy. They usually have an exercise routine apart from the sport designed to build and improve their body. They may seek a trainer to design exercises intended to improve strength, stamina, or balance and then push them in ways they would not normally push themselves. That trainer may have absolutely no experience with his or her client's sport, but can still be quite effective in designing an exercise program because the trainer is focused on building basic skills useful for any athlete in any sport. Education is similar: it is exercise for the mind. A teacher's job is like that of a physical trainer. A teacher develops exercises intended to improve communication skills, critical- thinking skills, and problem-solving skills. Different courses are like different types of fitness equipment. Some courses may focus more on communication through writing papers and through discussion and debate. Other courses, such as mathematics, focus more on critical thinking and problem solving.

Another similarity is that physical exercise must be challenging for your body to improve. Similarly, mental exercise must be challenging for the mind to improve. Expect course assignments to challenge you and push you mentally in ways you wouldn't push yourself. That's the best way to grow. So as you think about the courses you are taking and the assignments in those courses, remember the bigger picture of what you are developing: your mind. When you are writing papers, responding to questions, analyzing data, and solving problems, you are developing skills important to life and any career out there.

#### What Do I Need to Do to Succeed?

- Adequate Time To succeed, you must have adequate time and be willing to use that time to perform whatever is necessary. To determine if you have adequate time, use the following guide.
  - Step 1. Calculate your work hours per week.
  - Step 2. Calculate the number of hours in class each week.
  - **Step 3.** Calculate the number of hours required for study by doubling the number of hours you spend in class.
  - **Step 4.** Add the number of hours from steps 1–3.

Adequate time: If the total number of hours is below 60, then you have adequate time.

**Inadequate time:** If the total number of hours is 60 or more, then you do not have adequate time. You may be able to hang in there for a while, but eventually, you will find yourself overwhelmed and unable to fulfill all of your obligations. Remember, the above calculations do not consider other likely elements of life such as a commute, family, and recreation. The wise thing to do is cut back on work hours or drop some courses.

Assuming you have adequate time available, choosing to use that time to perform whatever is necessary depends on your attitude, commitment, and self-discipline.

✓ Positive Attitude We do not always get to choose our circumstances, but we do get to choose our reaction and behavior. A positive attitude is choosing to be cheerful, hopeful, and encouraging no matter the situation. A benefit of a positive attitude is that it tends to encourage people around you. As a result, they are more likely to want to help you achieve your goal. A negative attitude, on the other hand, tends to discourage people around you. As a result, they are less likely to want to help you. The best way to maintain a positive attitude is to keep life in perspective, recognizing that difficulties and setbacks are temporary.

- Commitment Commitment means binding yourself to a course of action. Remember, expect difficulties and setbacks, but don't give up. That's why a positive attitude is important; it helps you stay committed in the face of difficulty.
- Self-Discipline Self-discipline is choosing to do what needs to be done—even when you don't feel like it. In pursuing a goal, it is normal to get distracted or tired. It is at those times that your positive attitude and commitment to the goal help you discipline yourself to stay on task.

Thomas Edison, inventor of the lightbulb, provides an excellent example of all of these principles. Edison tried over 2000 different combinations of materials for the filament before he found a successful combination. When asked about all his failed attempts, Edison replied, "I didn't fail once, I invented the lightbulb. It was just a 2000-step process." He also said, "Our greatest weakness lies in giving up. The most certain way to succeed is always to try just one more time." In those two quotes, we see a person who obviously had time to try 2000 experiments, had a positive attitude about the setbacks, never gave up, and had the self-discipline to keep working.

#### **Behaviors of Strong Students and Weak Students**

The four requirements for success can be translated into behaviors. The following table compares the typical behaviors of strong students with the typical behaviors of weak students.

Strong Students	Weak Students
<ul> <li>are relaxed, patient, and work carefully.</li> <li>almost always arrive on time and leave the classroom only in an emergency.</li> <li>sit as close to the front as possible.</li> <li>pay attention to instruction.</li> <li>use courteous and respectful language, encourage others, make positive comments, are cheerful and friendly.</li> </ul>	<ul> <li>are rushed, impatient, and hurry through work.</li> <li>often arrive late and often leave class to "take a break."</li> <li>sit as far away from the front as possible.</li> <li>ignore instruction, chit-chat, draw, fidget, etc.</li> <li>use disrespectful language, discourage others, make negative comments, are grumpy and unfriendly.</li> <li>Examples of unacceptable language include: "I hate this stuff!" (or even worse!) "Are we doing anything important today?" "Can we leave early?"</li> </ul>
• ask appropriate questions and answer instructor's questions during class.	<ul> <li>avoid asking questions and rarely answer instructor's questions in class.</li> </ul>
• take a lot of notes, have organized notebooks, seek out and use study strategies	<ul> <li>take few notes, have disorganized notebooks, do not use study strategies</li> </ul>
• begin assignments promptly and manage time wisely and almost always complete assignments on time.	• procrastinate, manage time poorly, and often complete as- signments late.
<ul> <li>label assignments properly and show all work neatly.</li> </ul>	<ul> <li>show little or no work and write sloppily.</li> </ul>
• read and work ahead.	<ul> <li>rarely read or work ahead.</li> </ul>
• contact instructors outside of class for help, and use ad- ditional resources such as study guides, solutions manuals, computer aids, videos, and tutorial services.	• avoid contacting instructors outside of class and rarely use additional resources available.

Assuming you have the prerequisites for success and understand the behaviors of a good student, our next step is to develop two major tools for success:

- 1. **Learning Style:** Complete the Learning Styles Inventory to determine how you tend to learn.
- 2. **The Study System:** This system describes a way to organize your notebook, take notes, and create study tools to complement your learning style. We've seen students transform their mathematics grades from D's and F's to A's and B's by using the Study System that follows.

# **Learning Styles Inventory**

#### What Is Your Personal Learning Style?

A learning style is the way in which a person processes new information. Knowing your learning style can help you make choices in the way you focus on and study new material. Below are 15 statements that will help you assess your learning style. After reading each statement, rate your response to the statement using the scale below. There are no right or wrong answers.

3 =Often applies 2 =Sometimes applies 1 =Never or almost never applies

 1. I remember information better if I write it down or draw a picture of it.
 2. I remember things better when I hear them instead of just reading or seeing them.
 3. When I receive something that has to be assembled, I just start doing it. I don't read the directions.
 4. If I am taking a test, I can visualize the page of text or lecture notes where the answer is located.
 5. I would rather have the professor explain a graph, chart, or diagram to me instead of just showing it to me.
 6. When learning new things, I want to do it rather than hear about it.
 7. I would rather have the instructor write the information on the board or overhead instead of just lecturing.
 8. I would rather listen to an audiobook than read the book.
 <ul><li>8. I would rather listen to an audiobook than read the book.</li><li>9. I enjoy making things, putting things together, and working with my hands.</li></ul>
 <ul> <li>8. I would rather listen to an audiobook than read the book.</li> <li>9. I enjoy making things, putting things together, and working with my hands.</li> <li>10. I am able to conceptualize quickly and visualize information.</li> </ul>
 <ul> <li>8. I would rather listen to an audiobook than read the book.</li> <li>9. I enjoy making things, putting things together, and working with my hands.</li> <li>10. I am able to conceptualize quickly and visualize information.</li> <li>11. I learn best by hearing words.</li> </ul>
 <ul> <li>8. I would rather listen to an audiobook than read the book.</li> <li>9. I enjoy making things, putting things together, and working with my hands.</li> <li>10. I am able to conceptualize quickly and visualize information.</li> <li>11. I learn best by hearing words.</li> <li>12. I have been called hyperactive by my parents, spouse, partner, or professor.</li> </ul>
<ul> <li>8. I would rather listen to an audiobook than read the book.</li> <li>9. I enjoy making things, putting things together, and working with my hands.</li> <li>10. I am able to conceptualize quickly and visualize information.</li> <li>11. I learn best by hearing words.</li> <li>12. I have been called hyperactive by my parents, spouse, partner, or professor.</li> <li>13. I have no trouble reading maps, charts, or diagrams.</li> </ul>
<ol> <li>8. I would rather listen to an audiobook than read the book.</li> <li>9. I enjoy making things, putting things together, and working with my hands.</li> <li>10. I am able to conceptualize quickly and visualize information.</li> <li>11. I learn best by hearing words.</li> <li>12. I have been called hyperactive by my parents, spouse, partner, or professor.</li> <li>13. I have no trouble reading maps, charts, or diagrams.</li> <li>14. I can usually pick up on small sounds such as bells, crickets, frogs or on distant sounds such as train whistles.</li> </ol>

Write your score for each statement beside the appropriate statement number below. Then add the scores in each column to get a total score for that column.



The largest total of the three columns indicates your dominant learning style.



**Visual learners** learn best by seeing. If this is your dominant learning style, then you should focus on learning strategies that involve seeing. The color coding in the study system (see page xvii–xviii) will be especially important. The same color coding is used in the text. Draw diagrams, arrows, and pictures in your notes to help you see what is happening. Reading your notes, study sheets, and text repeatedly will be an important strategy.



Auditory learners learn best by hearing. If this is your dominant learning style, then you should use learning strategies that involve hearing. After getting permission from your instructor, bring a recorder to class to record the discussion. When you study your notes, play back the recording. Also, when you learn rules, say the rule over and over. As you work problems, say the rule before you do the problem. You may also find the videotapes to be beneficial because you can hear explanations of problems taken from the text.



**Tactile (also known as kinesthetic) learners** learn best by touching or doing. If this is your dominant learning style, you should use learning strategies that involve doing. Doing a lot of practice problems will be important. Make use of the Your Turn exercises in the text. These are designed to give you an opportunity to do problems that are similar to the examples as soon as a topic is discussed. Writing out your study sheets and doing your practice tests repeatedly will be important strategies for you.

Note that the study system developed in this text is for all learners. Your learning style will help you decide what aspects and strategies in the study system to focus on, but being predominantly an auditory learner does not mean that you shouldn't read the textbook, do a lot of practice problems, or use the color-coding system in your notes. Auditory learners can benefit from seeing and doing, and tactile learners can benefit from seeing and doing, and tactile learners can benefit from seeing and hearing. In other words, do not use your dominant learning style as a reason for not doing things that are beneficial to the learning process. Also remember that the Learning Strategy boxes presented throughout the text provide tips to help you use your personal learning style to your advantage.

Adapted from Sherfield, Robert M.; Moody, Patricia G., Cornerstone: Creating Success Through Positive Change, 6th Edition. © 2011. Adapted by permission of Pearson Education, Inc. Upper Saddle River, NJ.

# **The Carson Math Study System**

Organize the notebook into four parts using dividers shown:



#### Notes

- Use a color code: **red** for definitions, **blue** for rules or procedures, and **pencil** for all examples and other notes.
- Begin notes for each class on a new page (front and back for that day is okay). Include a topic title or section number and the date on each page.
- Try to write your instructor's spoken explanations along with the things he or she writes on the board.
- Mark examples your instructor emphasizes in some way to give them a higher priority. These problems often appear on quizzes and tests.
- Write warnings your instructor discusses about a particular situation.
- Include common errors that your instructor illustrates, but mark them clearly as errors so that you do not mistake them for correct.
- To speed note taking, eliminate unnecessary words such as *the* and use codes for common words such as + for *and* and  $\therefore$  for *therefore*. Also, instead of writing complete definitions, rules, or procedures, write the first few words and place the page reference from the text so that you can copy from the text later.

#### Sample Notes with Color Code



#### Homework

This section of the notebook contains all homework. Use the following guidelines whether your assignments are from the textbook, a handout, or a computer program such as MyMathLab or MathXL.

- Use pencil so that mistakes can be erased (scratching through mistakes is messy and should be avoided).
- Label according to your instructor's requirements. Usually, at least include your name, the date, and the assignment title. It is also wise to write the assigned problems at the top as they were given. For example, if your instructor writes "Section 1.5 #1–15 odd," write it that way at the top. Labeling each assignment with this much detail shows that you take the assignment seriously and leaves no doubt about what you interpreted the assignment to be.
- For each problem you solve, write the problem number and show all solution steps neatly.

Why do I need to show work and write all the steps? Isn't the right answer all that's needed?

- Mathematics is not just about getting correct answers. You really learn mathematics when you organize your thoughts and present those thoughts clearly using mathematical language.
- You can arrive at correct answers with incorrect thinking. Showing your work allows your instructor to verify that you are using correct procedures to arrive at your answers.
- Having a labeled, well-organized, and neat hard copy is a good study tool for exams.

### What if I submit my answers in MyMathLab or MathXL? Do I still need to show work?

Think of MyMathlab or MathXL as a personal tutor who provides the exercises, offers guided assistance, and checks your answers before you submit the assignment. For the same reasons as those listed above, you should still create a neatly written hard copy of your solutions, even if your instructor does not check the work. Following are some additional reasons to show your work when submitting answers in MyMathlab or MathXL.

- If you have difficulties that are unresolved by the program, you can show your instructor. Without the written work, your instructor cannot see your thinking.
- If you have a correct answer but have difficulty entering that correct answer, you have a record of it and can show your instructor. If correct, your instructor can override the score.

### Sample Homework: Simplifying Expressions or Solving Equations

Suppose you are given the following exercise:

For Exercises 1–30 simplify.

**1.**  $5^2 + 3 \cdot 4 - 7$ 

Your homework should look something like the following:



#### Sample Homework: Solving Application Problems

Example: Suppose you are given the following two problems.

For Exercises 1 and 2, solve.

- **1.** Find the area of a circle with a diameter of 10 feet.
- **2.** Two cars are traveling toward each other on the same highway. One car is traveling 65 miles per hour, and the other is traveling at 60 miles per hour. If the two cars are 20 miles apart, how long will it be until they meet?

Your homework should look something like the following:

		Section # Homewor	rk				10/6
	-0	#1 — 15 odd	If applicable, draw a picture				
16 Ale 1 1.			or table.				
a formula, v	on require vrite the	$1. A = \pi r^2$	2.	l ra	ite I time	; 10	listance
formula.			car	•1 65	mph t		65t
		10  ft. $r = 1/2$	(10 ft.) car	2 60	mph t		60t
		r = 5 ft.			,		
			cai	- 1	car 2		total
			distai	1ce +	distance	=	= distance
		$A = \pi (5  \text{ft.})^2$	65	5t +	60t	=	= 20
		$A = \pi (25  \text{ft.}^2)$	Translate to	an	125t	=	= 20
	$\bigcirc$	$A = 25\pi  \text{ft.}^2$	equation. T	hen	<u>125t</u>	=	= <u>20</u>
		A ≈ 25 (3.14) f	t. <sup>2</sup> show all so	lution	125		125
		$A \approx 78.5  \text{ft.}^2$	steps.		t	=	= 0.16
			Th	e cars n	neet in 0.1	6 h	our.
		Anow	or the question				
		Answ	er the question.				

#### Sample Homework: Graphing

Example: Suppose you are given the following two problems.

For Exercises 1 and 2, graph the equation.

**1.** y = 2x - 3 **2.** y = -2x + 1

Your homework should look something like the following:



#### **Study Materials**

This section of the notebook contains three types of study materials for each chapter. **Study Material 1: The Study Sheet** A study sheet contains *every* rule or procedure in the current chapter.

	Use the ch summary a	apter at the end	of					
	each chap	ter as a g	uide.	Chapter # Study S	heets	9/10	lf you a	ire a visual
	Write each ru procedure st They are in b notes and in	Ile or udied. Iue in you the text.	ir .	To graph a number on a number line, draw a dot on the mark for the number:	<i>ex) Graph</i> −2, ←		or tacti include exampl illustrat the rule proced	le learner, a key le to e what or ure says.
				number is positive.	ex)  7  = 7			
Inc hel pro Fo lea po jing	lude anything ps you remem ocedures and o r example, aud rners might w ems, rhymes, gles, as showr	that hber the rules. ditory rite or here.		To add same sign, Add and keep the same sign With different signs, subtract Keep the greater value's sign Add this, subtract that Can't you read the signs? Can't you read the signs?	ex) 7+5=12 ex) -7+(-5)= ex) -7+5=- ex) 7+(-5)=.	= - 12 2 2		

**Study Material 2: The Practice Test** If your instructor gives you a practice test, proceed to the discussion of creating a game plan.

If your instructor does not give you a practice test, use your notes to create your own practice test from the examples given in class. Include only the instructions and the problem, not the solutions. The following sample practice test was created from examples in the notes for Chapter 2 in a prealgebra course.

		Chapter # Pract	iice Test	10/10	
		For # 1 and 2, graph on a number	line.		For each example in
		1. 4			your notes, write the
		2. –3			problem but not the
					solution.
		For #3 and 4, simplify.		, ,	
After working throug	gh	3. [-8]			
the practice test, us	e	4.9			
your solutions.					
,		For #5 and 6, find the additive inv	erse of the number.		
		5. 13			
	$\bigcirc$	6. – 15			
		For #7 – 10, add.			
		7. 13 + (-9)	8. –	20+(-6)	
		9. – 15 + 8	10.	3+(-24)	
				. ,	

**Study Material 3: The Game Plan** The game plan refines the study process further. It is your plan for the test based on the practice test. For each problem on your practice test, write the definition, rule, or procedure used to solve the problem.

The sample shown gives the rule or procedure used to solve each problem on the preceding sample practice test. The rules and procedures came from the sample study sheet.

		Chapter # Game Plan	9/10	
Multiple prob use the same procedure ca grouped toge	lems that rule or n be ther.	<ul> <li>#1 and 2: Draw a dot on the mark for the number.</li> <li>#3 and 4: The absolute value of a positive number</li> <li>is a positive number.</li> </ul>	Write the procedu solve the on the p	e rule or re used to e problems ractice test.
		The absolute value of a negative number is a positive number: The absolute value of 0 is 0.		-
	0	#5 and 6: Additive inverses are numbers whose sum is 0.		-
		#7 – 10: To add same sign, Add and keep the same sign With different signs Subtract and keep the greater value's sign Add this subtract that		-
		Raa unis, subtract that Can't you read the signs? Can't you read the signs?		-

#### **Quizzes/Tests**

Archive all returned quizzes and tests in this section of the notebook.

- Midterm and final exam questions are often taken from the quizzes and tests, so they make excellent study tools for those cumulative exams.
- Keeping all graded quizzes and tests offers a backup system in the unlikely event your instructor should lose any of your scores.
- If a dispute arises about a particular score, you have the graded test to show your instructor.

	Chapter # Quiz	10/10
	For 1-4, simplify.	
	1.  −8  =8 ✓	$ \begin{array}{c} (4/4 = 100\%)\\ Nice\\ work! \end{array} $
	2.  9   = 9 🗸	
	3.−15+5 (=−10) ✓	
	4. −8+ −6 = −14) ✓	
0		

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### $\mathsf{C} \mathsf{H} \mathsf{A} \mathsf{P} \mathsf{T} \mathsf{E} \mathsf{R}$

### Foundations of Algebra

#### **Chapter Overview**

This chapter is a review of arithmetic, which is the foundation of algebra. More specifically, we review the following topics:

- ► Number sets.
- ▶ Properties of real numbers.
- ► Simplifying expressions.
- ► Operations of arithmetic.
- ► Evaluating expressions.

This review is by no means a complete instruction of arithmetic. Because a solid foundation is important, if you encounter a topic in this chapter that is a particular weakness for you, consult with your instructor or other more complete sources for extra practice.

- 1.1 Number Sets and the Structure of Algebra
- 1.2 Fractions
- **1.3** Adding and Subtracting Real Numbers; Properties of Real Numbers
- **1.4** Multiplying and Dividing Real Numbers; Properties of Real Numbers
- **1.5** Exponents, Roots, and Order of Operations
- **1.6** Translating Word Phrases to Expressions
- **1.7** Evaluating and Rewriting Expressions

# 1.1 Number Sets and the Structure of Algebra

#### **Objectives**

- 1 Understand the structure of algebra.
- 2 Classify number sets.
- **3** Graph rational numbers on a number line.
- 4 Determine the absolute value of a number.
- 5 Compare numbers.

#### Warm-up Refer to the To the Student Section on p. xiii.

- **1.** What are the four sections of the notebook?
- **2.** What is the color code for notes?
- 3. Complete the Learning Style Inventory. What is your learning style?

#### **Objective 1** Understand the structure of algebra.

Learning mathematics is like learning a language. When we learn a language, we must learn the alphabet, vocabulary, and sentence structure. Similarly, mathematics has its own alphabet, vocabulary, and sentence structure. In this section, we begin the development of the foundation of algebra with an overview of its components and structure. The basic components are **variables** and **constants**.

**Definitions Variable:** A symbol that can vary in value. **Constant:** A symbol that does not vary in value.

Variables are usually letters of the alphabet such as *x* or *y*. Usually, constants are symbols for numbers such as 1, 2,  $\frac{3}{4}$ , and 6.74. However, constants can sometimes be symbols such as *e* or the Greek letter  $\pi$ , each having special numeric values. Variables and constants are used to make **expressions**, **equations**, and **inequalities**.

#### Learning Strategy

It is important to sit up front in class because it helps you avoid distractions and makes it easier to hear any hints the professor might be giving.

–Jason J.

**Definition Expression:** A constant, a variable, or any combination of constants, variables, and arithmetic operations that describes a calculation.

**Examples of expressions:** 

$$2 + 6 \quad 4x - 5 \quad \frac{1}{3}\pi r^2 h$$

Definition Equation: A mathematical relationship that contains an equal sign.

**Examples of equations:** 

$$2 + 6 = 8 \quad 4x - 5 = 12 \quad V = \frac{1}{3}\pi r^2 h$$

**Connection** Think of expressions as phrases and equations as complete sentences. The expression 2 + 6 is read "two plus six," which is not a complete sentence. The equation 2 + 6 = 8 is read "two plus six is eight." Notice that the equal sign translates to the verb *is*, which makes the sentence a complete sentence.

Definition Inequality: A mathematical relationship that contains an inequality symbol

 $(\neq, <, >, \leq, \text{or} \ge).$ 

Answers to Warm-up

- 1. notes, homework, study materials, graded work
- **2.** red = definitions; blue = rules/procedures; pencil = all other notes
- 3. Answers may vary.

Symbolic Form	Translation
8 ≠ 3	Eight is not equal to three.
5 < 7	Five is less than seven.
7 > 5	Seven is greater than five.
$x \leq 3$	x is less than or equal to three.
$y \ge 2$	y is greater than or equal to two.

The algebra pyramid shown illustrates how variables, constants, expressions, equations, and inequalities relate. At the foundation of algebra and our pyramid are constants and variables, which are used to build expressions, which in turn are used to build equations and inequalities.



#### Objective 2 Classify number sets.

Numbers can be placed into different categories using sets.

**Definition Set:** A collection of objects.

Braces are used to indicate a set. For example, the set containing the numbers 1, 2, 3, and 4 is written {1, 2, 3, 4}. The numbers 1, 2, 3, and 4 are called the *members* or *elements* of this set.

#### **Procedure** Writing Sets

To write a set, write the members or elements of the set separated by commas within braces,  $\{ \}$ .

**Example 1** Write the set containing the first five letters of the alphabet.

**Answer:** {A, B, C, D, E}

**Your Turn 1** Write the set containing the first four months of the year.

Numbers are classified using number sets. The set of *natural numbers* contains the counting numbers 1, 2, 3, 4, ... and is written  $\{1, 2, 3, ...\}$ . The three dots are an *ellipsis* and indicate that the numbers continue forever in the same pattern. The set of *whole numbers* contains all of the natural numbers and 0 and is written  $\{0, 1, 2, 3, ...\}$ .



Developing a good study system and understanding how you best learn is essential to academic success. Make sure you familiarize yourself with the study system outlined in the To the Student section at the beginning of the text. Also take a moment to complete the Learning Styles Inventory found at the end of that section to discover your personal learning style. In these Learning Strategy boxes, we offer tips and suggestions on how to connect the study system and your learning style to help you be successful in the course.

Answer to Your Turn 1 {January, February, March, April} The set of *integers* contains all of the whole numbers and the opposite (or negative) of every natural number and is written  $\{ \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \}$ . Notice that the integers continue forever in both directions.

A number line is often useful in mathematics. The following number line is marked with the integers. Note that the positive numbers are to the right of 0 and the negative numbers are to the left.

									-
<u> </u>									
-4	1-3	3 - 2	2 -	1 (	) (	1 2	2 3	3 4	ŀ

Although we can mark and view only a portion of the number line, the arrows at the ends indicate that the line and the numbers on it continue forever in both directions. There are other types of numbers that include the integers and all numbers between them. If we traveled along the number line forever in both directions, we would encounter every number in the set of *real numbers*.

Some of the real numbers that are not integers are the **rational numbers** that contain every real number that can be expressed as a ratio of integers.

Definition Rational number: Any real number that can be expressed in the form

 $\frac{a}{b}$ , where *a* and *b* are integers and  $b \neq 0$ .

For example,  $\frac{3}{4}$  is a rational number because 3 and 4 are integers. Note that numbers such as 0.75 and 75% are also rational numbers because they can be written as  $\frac{3}{4}$ . It

is important to realize that the definition of a rational number does not state that the number *must* be expressed in the form  $\frac{a}{b}$ . Rather, if a number *can* be expressed in the

form  $\frac{a}{b}$ , it is a rational number. For example, the number 5 is a rational number because it can be expressed as  $\frac{5}{1}$ .

**Example 2** Determine whether the given number is a rational number.

**a.** 
$$\frac{2}{3}$$

Answer: Yes, because 2 and 3 are integers.

**b.** 0.4

**Answer:** Yes, 0.4 is a rational number because it can be expressed as  $\frac{4}{10}$  and 4 and 10 are integers.

**c.**  $0.\overline{6}$ 

**Answer:** A bar written over a decimal digit indicates that the digit repeats without end. So  $0.\overline{6} = 0.66666 \dots$ , and we say that these decimal numbers are nonterminating decimal numbers. Usually, we encounter these numbers as quotients in certain division problems, such as when we write certain fractions as decimals. In this case,

 $0.\overline{6}$  is the decimal equivalent of  $\frac{2}{3}$ . Because  $0.\overline{6}$  can be expressed as the fraction  $\frac{2}{3}$ , it is a rational number.

**d.** 3

**Answer:** Yes, because 3 can be thought of as  $\frac{3}{1}$ ,  $\frac{6}{2}$ ,  $\frac{9}{3}$ , etc. All integers are rational numbers.

**Note** In the definition for a rational number, the notation  $b \neq 0$  is important because if the denominator *b* were to equal 0, the fraction would be undefined. The reason it is undefined will be explained later.

Note All nonterminating decimal numbers with repeating digits are rational numbers because they can be expressed as fractions with integers in both the numerator and denominator.

**Your Turn 2** Determine whether the given number is a rational number.

**a.** 
$$\frac{4}{7}$$
 **b.** 0.56 **c.**  $0.\overline{2}$  **d.** -7

Not all numbers can be expressed as a ratio of integers. One such number is  $\pi$  (pronounced "pie"). Because the exact value of  $\pi$  cannot be expressed as a ratio of integers, it is categorized as an **irrational number**.

**Definition Irrational number:** Any real number that is not rational.

Some other irrational numbers are  $\sqrt{2}$  and  $\sqrt{3}$ . (Square roots are explained in more detail in Section 1.5.) Because an irrational number cannot be written as a ratio of integers, if a calculation involves an irrational number, we must leave it in symbolic form or use a rational number approximation. We can approximate  $\pi$  with rational

numbers such as 3.14 and  $\frac{22}{7}$ . Any decimal representation of an irrational number is a nonrepeating, nonterminating decimal number. For example, 0.1010010001 . . . is

Every **real number** is either rational or irrational.

irrational.

Definition Real number: Any rational or irrational number.

The following figure shows how the number sets relate in the real number system.

#### **Of Interest**

There are symbols for the number sets. In particular, the set of real numbers is often written as  $\mathbb{R}$  or  $\mathcal{R}$ .

Real Num	bers
<b>Rational Numbers:</b> Real numbers that can be expressed in the form $\frac{a}{b}$ , where <i>a</i> and <i>b</i> are integers and $b \neq 0$ , such as $-4\frac{3}{4}$ , $-\frac{2}{3}$ , 0.018, $0.\overline{3}$ , and $\frac{5}{8}$ . Integers:, $-3$ , $-2$ , $-1$ , $0$ , $1$ , $2$ , $3$ , Whole Numbers: 0, 1, 2, 3, Natural Numbers: 1, 2, 3,	<b>Irrational Numbers:</b> Any real number that is not rational, such as $-\sqrt{2}$ , $-\sqrt{3}$ , $\sqrt{0.8}$ , and $\pi$ .

**Objective 3** Graph rational numbers on a number line.

Number lines can be useful tools when comparing numbers or solving certain arithmetic problems. Let's review how to graph a number on a number line.



**a.**  $1\frac{3}{5}$ 

**Solution:** The number  $1\frac{3}{5}$  is located  $\frac{3}{5}$  of the way between 1 and 2, so we divide the space between 1 and 2 into 5 equal divisions and place a dot on the 3rd mark.



Answers to Your Turn 2 a. yes b. yes c. yes d. yes

**b.** 
$$-2.56$$

**Solution:** Because -2.56 means  $-2\frac{56}{100}$ , we could divide the space between -2 and -3 into 100 divisions and count to the 56th mark. Because this is tedious to do, we gradually use smaller and smaller sections of the number line to graph the number.



**Your Turn 3** Graph on a number line.



Objective 4 Determine the absolute value of a number.

The word *value* indicates how much something is worth, or its magnitude. In mathematics, **absolute value** indicates the magnitude of a number.

Definition Absolute value: A given number's distance from 0 on a number line.

For example, the absolute value of 5 is 5 because the number 5 is 5 units from 0 on a number line. Likewise, the absolute value of -5 is 5 because -5 is also 5 units from 0 on a number line.



In the To the Student section, we suggest that when taking notes, you use a red pen for definitions and a blue pen for rules and procedures. Notice that we have used those colors in the design of the text to connect with your notes.



The absolute value of a number n is written |n|. The examples just mentioned translate this way:



What about the absolute value of 0? Because there are no units between 0 and itself, the absolute value of 0 is 0.

#### Rule Absolute Value

The absolute value of every real number is either positive or 0.



Answers to Your Turn 3

 $\rightarrow Example 4 Simplify.$ a. |-4.5|Answer: |-4.5| = 4.5 because -4.5 is 4.5 units from 0 on a number line.

**b.** 
$$\left|\frac{3}{8}\right|$$
  
**Answer:**  $\left|\frac{3}{8}\right| = \frac{3}{8}$  because  $\frac{3}{8}$  is  $\frac{3}{8}$  of a unit from 0 on a number line  
**Your Turn 4** Simplify.  
**a.**  $\left|-2\frac{4}{5}\right|$  **b.**  $|12.8|$ 

#### Objective 5 Compare numbers.

Number lines can also be used to determine which of two numbers is greater. Because numbers increase from left to right on a number line, the number farther to the right will be the greater of two numbers.

#### **Rule** Comparing Numbers

For any two real numbers *a* and *b*, *a* is greater than *b* if *a* is to the right of *b* on a number line. Equivalently, *b* is less than *a* if *b* is to the left of *a* on a number line.

Consider the following number line where we compare 2 and 8.



Because the number 8 is farther to the right on a number line than the number 2 is, we say that 8 is greater than 2, or in symbols, 8 > 2. Or we could say that because 2 is to the left of 8, 2 < 8.

**Example 5** Use =, <, or > to write a true statement.

**Answer:** 4 > -4 because 4 is farther to the right on a number line than -4 is.

**b.** -2.7 -2.5

**Answer:** -2.7 < -2.5 because -2.7 is farther to the left on a number line than -2.5 is.

c. 
$$\left|3\frac{5}{6}\right| = 3\frac{5}{6}$$
  
Answer:  $\left|3\frac{5}{6}\right| = 3\frac{5}{6}$  because the absolute value of  $3\frac{5}{6}$  is equal to  $3\frac{5}{6}$ .

**d.** |−0.7| −1.5

**Answer:** |-0.7| > -1.5 because the absolute value of -0.7 is equal to 0.7, which is farther to the right on a number line than -1.5 is.

**Your Turn 5** Use =, <, or > to write a true statement. 

 a. -15 > -21 b. -4.1 < 0 

 c.  $2\frac{5}{6} > 2\frac{1}{4}$  d. |-9.5| = 9.5 

 a. -15 -21 

 b. -4.1 0

 c.  $2\frac{5}{6} > 2\frac{1}{4}$  d. |-9.5| = 9.5 

#### Answers to Your Turn 4

**a.**  $2\frac{4}{5}$ **b.** 12.8





*Note:* Exercises marked with a **★** represent challenging exercises.

#### **Objective 1**

**Prep Exercise 1** Explain the difference between a constant and a variable.

**Prep Exercise 2** An expression is a constant, a variable, or any combination of constants, variables, and arithmetic operations that describes a(n) \_\_\_\_\_.

**Prep Exercise 3** An equation is a mathematical relationship that contains a(n) \_\_\_\_\_\_.

Prep Exercise 4 List the inequality symbols.

**Prep Exercise 5** Complete the name of each level of the algebra pyramid.



#### **Objective 2**

**Prep Exercise 6** To write a set, write the members or elements of the set separated by commas within \_\_\_\_\_\_.

For Exercises 1–10, write a set representing each description. See Example 1.

- 1. The days of the week
- 3. The vowels of the English alphabet
- 5. The natural numbers that are multiples of 5
- 7. The odd natural numbers greater than 7
- 9. The integers greater than or equal to -6 but less than -2

- **2.** The last ten letters of the English alphabet
- **4.** The states in the United States that do not share a border with any other state
- 6. The even natural numbers
- 8. The even integers greater than or equal to 16
- **10.** The integers greater than -2.1 and less than  $\frac{3}{4}$

#### Prep Exercise 7 Explain the difference between a rational number and an irrational number.

For Exercises 11–20, determine whether each number is a rational number or an irrational number. See Example 2.

11.	$-\frac{4}{5}$	<b>12.</b> $\frac{1}{4}$	<b>13.</b> 9		<b>14.</b> -12	15. $\pi$
16.	$\frac{\pi}{4}$	170.21	<b>18.</b> -0.8		<b>19.</b> 0.6	<b>20.</b> 0.13
Prep	Exercise 8 Every real	number is either	or	·		
For	Exercises 21–26, answe	r true or false.				
21.	Every rational numb	er is a real number.		<b>22.</b> Every 1	eal number is a rati	ional number.
23.	Every whole number	r is an integer.		<b>24.</b> Every 1	eal number is a nat	ural number.
25.	A number exists that	is both rational and irra	itional.	<b>26.</b> Zero is	a rational number.	
Obj	jective 3					
Ган	Freedom 27 24 grant		u lina Cas Fuar			
FOr		euch number on a number	r tine. See Exur	npie 5.		2
27.	$-\frac{5}{6}$	<b>28.</b> $5\frac{1}{2}$		<b>29.</b> $2\frac{3}{8}$		<b>30.</b> $-\frac{2}{5}$
31.	-3.5	<b>32.</b> 7.4		<b>33.</b> 2.45		<b>34.</b> -7.62
Obj	jective 4					
Prep	<b>Exercise 9</b> The absolu	ite value of every real nu	ımber is either		_ or	
For	Exercises 35–44, simpli	fy. See Example 4.				
35.	23	<b>36.</b>  6	<b>37.</b>  -2		<b>38.</b>  -8	<b>39.</b>  -5.7
40.	-4.5	<b>41.</b> $\left  -3\frac{1}{8} \right $	<b>42.</b> $\left 2\frac{3}{5}\right $		<b>43.</b>  0	<b>44.</b>  -67.8
Obj	jective 5					
<b>Prep</b> a nu	Exercise 10 When com Imber line is the greate	paring two numbers, the er number.	e number farth	er to the	on	
For	Exercises 45–72, use =	= , < , or > to write a t	rue statement.	See Example 5	5.	

**47.** 4 -3 **48.** -6 5 **49.** -7 -8

**46.** 2 7

**45.** 9 3